ISSN 2229-5518

# Designing a Multi-Level Controller in Aircraft Pitch Angle

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**Abstract**— Progress in aircraft designs heavily depend on Automatic Flight Control System (AFCS). In this work a fuzzy sliding mode controller is designed as a supervisory controller in aircraft pitch control systems and different control strategies to model a pitch controller based on design a pitch angle control in autopilot are invistigated. A performance evaluation based on time response specification between modern control Linear Quadratic Regulator (LQR) and multi-level intelligent controller (based on Fuzzy Logic Controller, FLC and Supervisory FLC, SFLC) for a pitch control system is presented. The performances of pitch control systems are analyzed based on common criteria of step's response in order to identify which control strategy delivers better performance with respect to the desired pitch angle. In this work, new approach, SFLC is presented to improve the performance of pure FLC in term of rising time and settling time. It is found that, LQR controller give the better performance compared to pure FLC, and SFLC gives the best performance.

**Index Terms**— Aircraft pitch control, Autopilot, Fuzzy logic, Linear Quadratic Regulator (LQR), Supervisory controller.

## 1. Introduction

Aircraft design has developed dramatically after Wright brothers through structures, materials, aerodynamics, propulsion and flight control using many of new techniques. Since the early days, the concept of AFCS has evolved from mechanical control systems to highly advanced automatic fly-by-wire flight control systems which can be found nowadays in military jets and civil airplane. All modern aircraft depend upon their flight control system to provide the handling qualities necessary for successful flight. Modern aircraft contain a variety of AFCS that help the flight crew in navigation, reduce pilot workload, stability and control augmentation and management of the airplane. For this situation an autopilot is designed that control the pitch of aircraft that can be used by the flight crew [1].

The autopilot is a main component within AFCS that can be capable of much very time intensive tasks, helping the pilot focus on the overall status of the aircraft and flight. Also good use of an autopilot helps automate the process of guiding and controlling the aircraft. Autopilots can automate different tasks, such as maintaining an altitude, climbing or descending to an assigned altitude, turning to and maintaining an assigned heading, intercepting a course, guiding the aircraft between waypoints that make up a route programmed into a flight management system, and flying a precision or no precision approach.

Designing an autopilot requires control system theory and knowledge of stability derivatives at different altitudes and Mach numbers for a given airplane [2]. Even today, many research efforts are made for the further development of these flight control systems in various aspects to control pitch of an aircraft for the purpose of flight stability and yet this research remains an open issue in the present and future works [3 to 5].

In this study, we propose fuzzy sliding mode control as a supervisory controller in a multi level control of aircraft pitch angle. To doing this, in the next section, we present aircraft dynamic modeling, then we develop design controllers, and finally we discuss simulation results.

## 2. Mathematical Modelling

Aircraft dynamic generally are nonlinear, time varying, and including uncertainty in parameters (such as aerodynamic). AFCS has been designed using linearized aircraft dynamic models at different flight conditions [6]. This study is developed to control the pitch angle of an aircraft in order to stabilize the system when the airplane nose is pitched up (down). The pitch control system is shown in Fig. 1, where $X_b$ and $Z_b$ represent the aerodynamic force components, $\theta$ , represent the orientation of aircraft pitch angle in the earth-axis system and $\delta_e$ elevator deflection angle [1]. Fig. 2 shows the forces, moments and velocity components in the body fixed coordinate of aircraft system. The aerodynamic moment components are represented as L, M and N. The terms p,q and r represent the angular rates about roll, pitch and yaw axis while the terms, u ,$v$ and $w$ represent the velocity components of roll, pitch and yaw axis.$\alpha$ and $\beta$ are angle of attack and sideslip angle respectively [1].

In this work, we use the data for JetStar General Aviation airplane from [1] in flight speed $u_0=340$ ft/sec and at 40000 ft. The longitudinal stability derivatives parameters are denoted in Table. 1.

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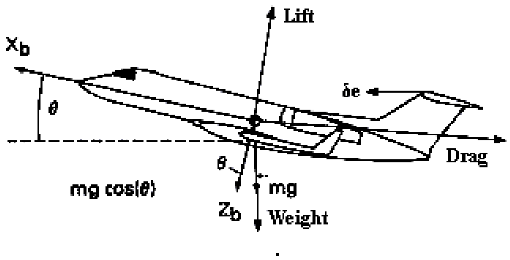


**Fig. 1**. Description of Pitch Control

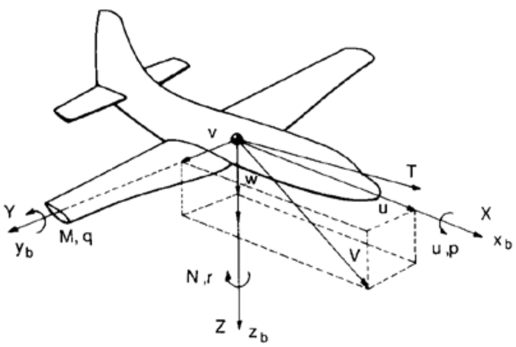


**Fig. 2**. Definition of Force, Moment and Velocity Components in a Body Fixed Coordinate.

**Table 1**
Longitudinal Deravatives Stability Parameters [1]

| Flight Condition | H= 40,000 ft ; M= 0.4 Components | | |
|---|---|---|---|
| Longitudinal Derivatives | X-Force ($S^{-1}$) | Z-Force ($S^{-1}$) | Pitching Moment ($FT^{-1}$) |
| Rolling velocities | $X_u=-0.045$ | $Z_u=-0.369$ | $M_u=0$ |
| Yawing velocities | $X_w=0.036$ | $Z_w=-2.02$ | $M_w=-0.05$ |
| | $X_{\dot{w}}=0$ | $Z_{\dot{w}}=0$ | $M_{\dot{w}}=-0.051$ |
| Angle of attack | $X_\alpha=0$ | $Z_\alpha=-355.42$ | $M_\alpha=-8.8$ |
| | $X_{\dot{\alpha}}=0$ | $Z_{\dot{\alpha}}=0$ | $M_{\dot{\alpha}}=-0.8976$ |
| Pitching rate | $X_q=0$ | $Z_q=0$ | $M_q=-2.05$ |
| Elevator deflection | $X_{\delta e}=0$ | $Z_{\delta e}=-28.15$ | $M_{\delta e}=-11.874$ |

In designing of pitch control system is assumed that aircraft is in steady state cruise (at constant altitude and Mach number) and the change in pitch angle does not change the speed of an aircraft under any circumstance. So, by this assumption, the drag and the thrust forces are canceled out and lift and weight, balance out each other. By writing the forces and moments acting on aircraft as shown in Fig. 1 and 2, the transfer function of pitch control system obtained [1] as:

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)}=\frac{-\left(M_{\delta e}+M_{\alpha}\frac{Z_{\delta e}}{u_0}\right)s-\left(M_{\alpha}\frac{Z_{\delta e}}{u_0}-M_{\delta e}\frac{Z_{\alpha}}{u_0}\right)}{s^3-\left(M_q+M_{\alpha}\frac{Z_{\alpha}}{u_0}\right)s^2+\left(Z_{\alpha}\frac{M_q}{u_0}-M_{\alpha}\right)s} \quad (1)$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)}=\frac{11.7304s+22.578}{s^3+4.9676s^2+12.941s}$$

## 3. METHODOLOGIES

It is shown that LQR controller work better than classical FLC [7]. In this work, we investigate on enhancing performance of FLC by designing of SFLC. In the following section, we describe in detail LQR, FLC and SFLC. Furthermore, a few of design specification have to be set to investigate the performance of both control strategies. In this work, four considerations have to be met which are rising time less than (3) second, settling time less than (5) second, percentage of overshot less than (10%) and steady state error less than 2% for controlling the pitch angle of 0.2 radian ($11.5^0$) [7].

### 3.1 Modern Control Based on LQR

LQR is a method in modern control theory that used state-space approach to analyze and control a system [8]. Using state space methods, it is relatively simple to work with a multi-output system. The configuration of this control system is shown in Fig.3.

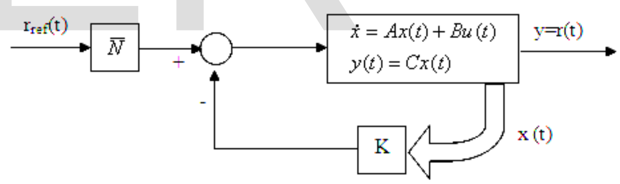


**Fig. 3.** Full-state feedback controller with reference input

In designing LQR controller, we use from "$lqr$" function in MATLAB to determine the value of the vector $K$, which determined the feedback control law. By choosing two parameter R=1 and Q=μ.$C^T$.C , the controller can be tuned by changing $\mu$, which is obtained:

$$R=1;\ Q=[0\ 0\ 0;\ 0\ 0\ 0;\ 0\ 0\ \mu]$$

$$K=lqr\ [A, B, Q, R] \quad (2)$$

In order to reduce steady state error of the system output, we use scale the reference input r(t) so that:

$$u=\bar{N}.r-K.X(t) \quad (3)$$

If μ is increased even higher, improvement to the response should be obtained even more, but for this case, the values of μ=500 is chosen, because it satisfied the design requirements while keep μ as small as possible. Consequently, by tuning the value of μ=500, the following values of matrix K are obtained.

$$K = \begin{bmatrix} -0.5704 & 1.6929 & 22.3607 \end{bmatrix}$$

For this controller design, the value of constant gain $\bar{N}$ , are found to be, 22.3607. Simulation results for the LQR controller is shown in Fig.4.

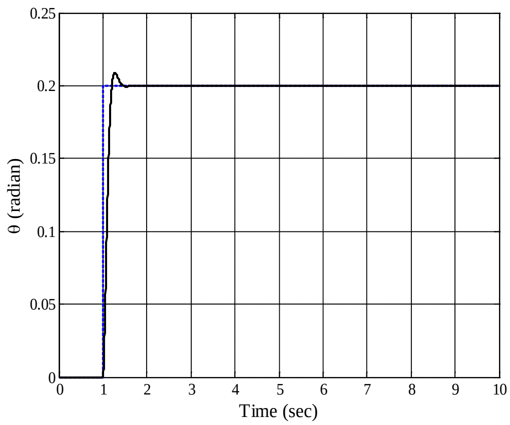


**Fig. 4.** Pitch angle response with LQR controller

### 3.2 Fuzzy Logic Controller

Fuzzy theory was initiated by Lotfi A. Zadeh in 1965 with his seminal paper "Fuzzy Set" (Zadeh, 1965). After that, the field of fuzzy systems and control has been making rapid progress in recent years by practical success of fuzzy control in consumer products and different industrial process control [9].

In this work, a two-level control system has been applied, where the first level FLC performs the main control action, and the second level Supervisory FLC has been added to give a good performance. The configuration of this suggested fuzzy controller is shown in Fig. 5.

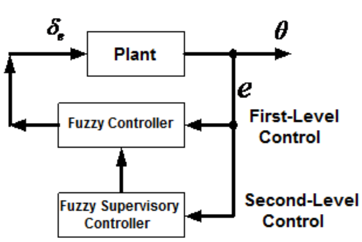


**Fig. 5.** Architecture of suggested FLC

#### 3.2.1 First –Level Fuzzy Logic Controller

The inputs to the FLC are the error (e), which measures the system performance and the rate at which the error changes (Δe) whereas the output is the change of the control signal ($\delta_e$) as shown in Fig.6.

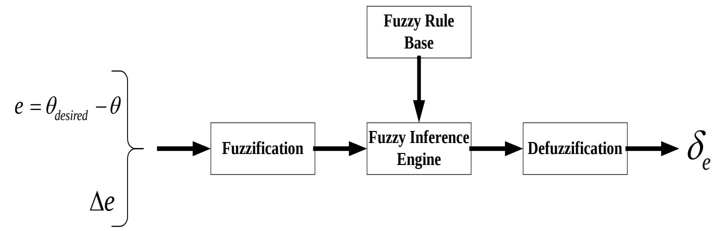


**Fig. 6.** Input and Output for FLC

The error (e) is computed by comparing the reference point (desired angle) with the plant output

$$e(t) = \theta_{desired}(t) - \theta(t)$$

The change of error (Δe) is generated by the derivation of the error:

$$\left( \Delta e(t) = \frac{de(t)}{dt} \right)$$

.

Fig. 7 shows the overall closed-loop system for FLC with the pitch control of an aircraft. Fuzzification involves the conversion of the input and output signals into a number of fuzzy represented values (fuzzy set). Each fuzzy set consists of three types membership function, which is negative (N), zero (Z) and positive (P). These are nine rules that have been utilized in designing the controller and the rule is defined in Table 2.

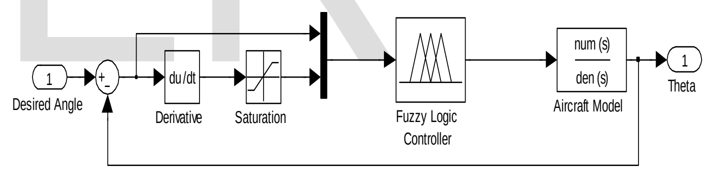


**Fig. 7.** FLC in feedback loop of pitch control

**TABLE 2**

RULES FOR THE FUZZY CONTROLLER

| Δe \ e | N | Z | P |
|---|---|---|---|
| **N** | N | N | P |
| **Z** | N | Z | P |
| **P** | N | P | P |

#### 3.2.2. Supervisory Controller

In many complex systems, the single loop control systems may not effectively achieve the control objectives and a multi level control structure turns out to be very helpful [9]. The main advantage of two level controls is that different controllers can be designed to achieve different objectives, so that each controller is simpler and performance improved. In this work, a fuzzy sliding mode controller is applied as a supervisory controller, in order to improving the controller performance. The Fig.8 shows the block diagram of supervisory controller.

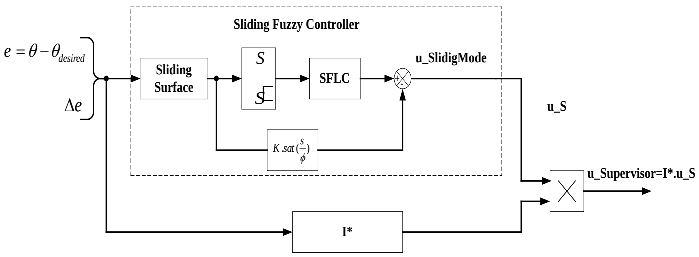


**Fig. 8.** Block Diagram of Supervisor FLC

The control command is:

$$\delta_e = u_{fuzzy} + I^*.u_{sliding\ mode} \qquad (4)$$

Where ($u_{fuzzy}$) is the output of FLC, which was defined in last section and ($I^*$) is switching algorithm.

### 3.2.2.1 Sliding mode Controller

The sliding surface is defined as [9]:

$$s = \left(\frac{d}{dt} + \lambda\right)^1 .e = \dot{e} + \lambda e \qquad (5)$$

Where: $e(t) = \theta(t) - \theta_{desired}(t)$, $\lambda$ is a positive constant and the constant factor $\phi$ defines the thickness of the layer, and $sat\left(\frac{s}{\phi}\right)$ is a saturation function that is as:

$$sat\left(\frac{s}{\phi}\right) = \begin{cases} -1 & \frac{s}{\phi} \le -1 \\ \frac{s}{\phi} & -1 < \frac{s}{\phi} \le 1 \\ +1 & \frac{s}{\phi} > 1 \end{cases} \qquad (6)$$

The value of ϕ is selected to be the thickness of the boundary layer (equals to 0.11 deg), and after some iteration the values of constants K and λ are selected to give a good performance, K=5 and λ=5. By applying these values, the steady state error has been appeared, and also an error is found at the interval [0, 1] which there is no refernce signal found. The indicator function (switching algorithm $I^*$) in equation (4) is defined as:

$$I^* = \begin{cases} 0 & |e| < a \\ \frac{|e| - a}{M_x - a} & a \le |e| < M_x \\ 1 & |e| \ge M_x \end{cases} \qquad (7)$$

Where $M_x$ is a positive constant, and is selected to give a desired performance (rise time and settling time), and $a$ is selected to be smaller than $\phi$. Fig.9 shows the simulation results of supervisory fuzzy controller to pitch control of aircraft.

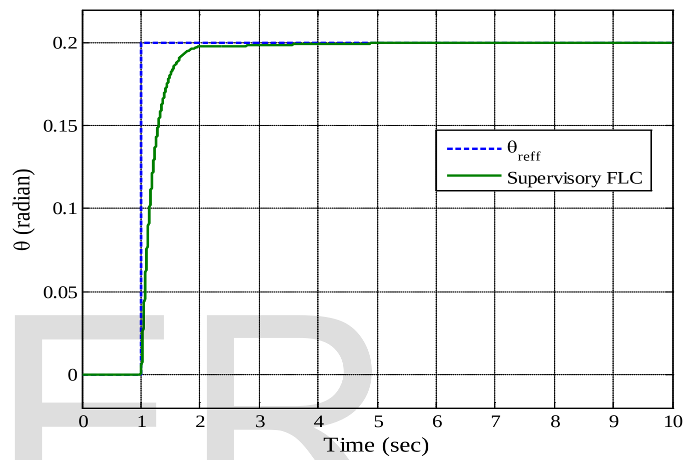


**Fig. 9.** Pitch angle response with Supervisory-FLC

For comparison of controller performance, the response for pitch control of an aircraft system using LQR, FLC and Supervisory-FLC are shown with overall response of both controllers in Fig. 10 .

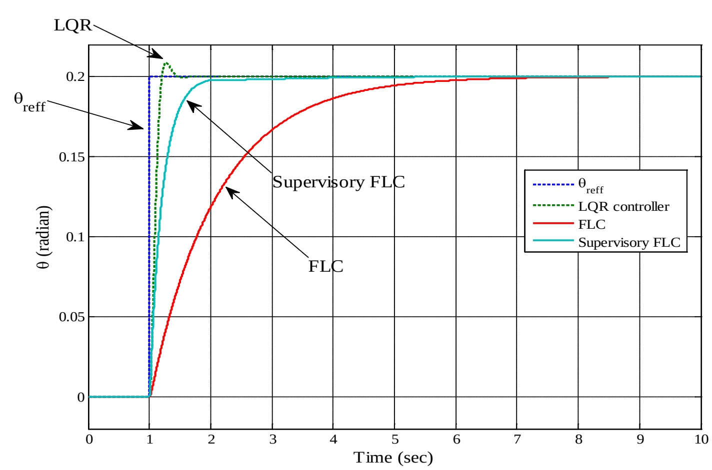


**Fig. 10.** Pitch angle response for LQR, FLC, and supervisory-FLC

ISSN 2229-5518

## 4. Discussion of results

In this section, the proposed control schemes and the corresponding results are presented. A unit step command is required in order for pitch angle to follow the reference value of 0.2 radian (11.5 degrees). The pitch control system with both LQR and supervisory fuzzy logic controller produced the response of pitch angle $(\theta)$. The system response with LQR is shown in Fig.4. The summary for the performance characteristics of the step response for the pitch angle between LQR, FLC and supervisory FLC is shown in Table 3 quantitatively.

**TABLE 3**
Summary Results for Pitch angle controller

| Response characteristic | Controller | | |
|---|---|---|---|
| | **LQR** | **FLC** | **SFLC** |
| Rising Time $T_r$ | 0.1323 | 2.4408 | 0.4671 |
| Settling Time $T_s$ | 0.1826 | 3.3497 | 0.6470 |
| Percent Overshoot (%OS) | 4.3474 | 0 | 0 |
| Steady-state Error $(e_{ss})$ (%) | 0 | 0.0493 | 0.0029 |

By referring to the Fig. 4 and Table .3, the results clearly demonstrate that LQR controller has the fastest response with the settling time of 0.1826 second and rising time of 0.1325 second. For the percent of overshoot (%OS), LQR has 4.35% which is met the desired requirement of controller design. Furthermore, the LQR controller tends to produce zero steady state error (Ess). This can be indicating that LQR controller can handle the effect of disturbances in the system.
The FLC provides good performance in term of percent overshoot that is 0%. As depicted from Fig.10, it can be observed that the pitch angle follows the reference value respectively. This controller is able to give a good response without produce any overshoot. The response is comparatively fast that give the settling time (Ts) about 3.3497 second and rise time (Tr) about 2.4408 second. The results also demonstrated that the steady state error (Ess) is 0.0493%. By referring to fig. 10, it is shown that SFLC provides good performance in term of percent overshoot that is 0%, so the pitch angle follows the reference value respectively. This controller is able to give a good response without produce any overshoot. The response is comparatively fast that give the settling time (Ts) about 0.6470 second and rise time (Tr) about 0.4671 second. The results also demonstrated that the steady state error (Ess) is 0.0029 %. As is shown in Figure 10, the results clearly shows that LQR controller has the best performance as compared to FLC and SFLC in term of rising time (Tr), settling time (Ts) and percent of steady state error (Ess). However, for the percent of overshoot (%OS) FLC and SFLC have the best range which is 0%. In addition, it is clear that the SFLC has better performance than FLC in all terms. Therefore, it can be concluded that the LQR controller provide higher ability in controlling the pitch angle as compared to the FLC, but the SFLC improves the FLC and gives very good performance without overshoot.

## 5. Conclusion

The validated model of pitch control of aircraft is very helpful in developing the control strategy for actual system. Pitch control of an aircraft is a system which requires a pitch controller to maintain the angle at it desired value. This can be achieved by reducing the error signal which is the difference between the output angle the desired angle. Three controllers, LQR, FLC and SFLC are successfully designed and presented. Based on the result and the analysis, a conclusion has been made that, the control approach of LQR, FLC and SFLC is capable on controlling the pitch angle of the aircraft system for value of 0.2 radian (11.5degree). Simulation and analysis results show that, LQR controller relatively give the better performance compared to FLC and SFLC in controlling the pitch angle of an aircraft system. In this work, new approach, depending on SFLC, is presented to improve the performance of pure FLC in term of rising time and settling time.

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